

Differentiation

$$1. f(x) = \frac{3x-1}{x+2}, \quad x \neq -2$$

from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{3x-1}{x+2} \quad f(x+h) = \frac{3(x+h)-1}{x+h+2}$$

$$= \frac{3x+3h-1}{x+h+2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{3x+3h-1}{x+h+2} - \frac{3x-1}{x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+2)(3x+3h-1) - (x+h+2)(3x-1)}{h(x+h+2)(x+2)}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 3xh - x + 6x + 6h - 2 - (3x^2 + 3xh + 6x - x + 2x)}{h(x+h+2)(x+2)}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 3xh - x + 6x + 6h - 2 - 3x^2 - 3xh - 6x + x + 2x}{h(x+h+2)(x+2)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{6h+h}{x^2 + 4x + 4 + xh + 2h} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{7h}{x^2 + 4x + 4 + xh + 2h} \right)$$

$$= \lim_{h \rightarrow 0} \frac{7}{x^2 + 4x + 4 + xh + 2h}$$

$$f'(x) = \frac{7}{x^2 + 4x + 4}$$

$$b) d) 3x^2 - 10x + 6$$

$$f'(x) = 6x - 10$$

$$b) 3\sqrt{x} - \frac{2}{x^2}$$

$$= 3x^{1/2} - 2x^{-2}$$

$$f'(x) = 3 \times \frac{1}{2} x^{-1/2} - 2(-2)x^{-3}$$

$$= \frac{3}{2} x^{-1/2} + 4x^{-3}$$

$$= \frac{3/2}{\sqrt{x}} + \frac{4}{x^3}$$

$$c) x(x+1)(x+2)$$

$$= (x^2+x)(x+2)$$

$$= x^3 + x^2 + 2x^2 + 2x$$

$$= x^3 + 3x^2 + 2x$$

$$f'(x) = 3x^2 + 6x + 2$$

$$d) \frac{x}{x+1}$$

$$= x(x+1)^{-1}$$

$$f'(x) = x(-1)(x+1)^{-2} + (x+1)^{-1}$$

$$= -x(x+1)^{-2} + (x+1)^{-1}$$

$$= \frac{-x}{(x+1)^2} + \frac{1}{(x+1)}$$

$$e) \sqrt{1+x^2}$$

$$= (1+x^2)^{1/2}$$

$$f'(x) = \frac{1}{2} (1+x^2)^{-1/2} \cdot 2x \quad \text{(using chain rule)}$$

$$= \frac{1}{2} \cdot 2x$$

$$= \frac{x}{\sqrt{1+x^2}}$$

$$8) \frac{dx}{\sqrt{x^2+7}}$$

$$= \frac{-dx}{-2x} = \frac{1}{2} \frac{dx}{x}$$

$$= \frac{1}{2} \ln(x^2+7)^{-1/2}$$

Using differentiation by parts $-\frac{3}{2}$

$$p'(x) = 2x(-\frac{1}{2})(x^2+7)^{-3/2} + 2(x^2+7)^{-1/2}$$

$$= -2x^2(x^2+7)^{-3/2} + 2(x^2+7)^{-1/2}$$

$$= \frac{-2x^2}{(\sqrt{x^2+7})^3} + \frac{2}{\sqrt{x^2+7}}$$

$$9) \int \frac{1-2x}{1+2x}$$

$$= \frac{(1-2x)(1+2x)^{1/2}}{(1+2x)^{3/2}}$$

By differentiation by parts

$$= \frac{1}{2} \left[\frac{(1-2x)}{(1+2x)^{1/2}} \right]$$

$$= \frac{1}{2} \left[\frac{(1+2x-2x-4x^2)}{(1+2x)^{1/2}} \right]$$

$$f'(x) = \frac{1}{2}$$

$$9) \int \frac{1-2x}{1+2x}$$

$$= \left[\frac{(1-2x)(1+2x)^{1/2}}{(1+2x)^{3/2}} \right]$$

$$= \left[\frac{(1+2x-2x-4x^2)}{(1+2x)^{1/2}} \right]$$

$$= \left[\frac{(1-4x^2)}{(1+2x)^{1/2}} \right] = \frac{1}{2} (-8x)$$

$$f(x) = \frac{1}{2} [1-4x^2]^{1/2} + C$$

Using chain rule

$$= -4x(1-4x^2)^{-1/2}$$

$$= \frac{-4x}{\sqrt{1-4x^2}}$$

$$h) \frac{5}{2+\sqrt{1+3x}}$$

$$= \frac{5}{2+(1+3x)^{1/2}}$$

$$= 5[2+(1+3x)^{1/2}]^{-1}$$

$$= 5[2^{-1} + (1+3x)^{-1/2}]$$

$$f'(x) = \frac{1}{5} \cdot 5 [(1+3x)^{-1/2} \cdot 3]$$

$$= \frac{5}{2} [3(1+3x)^{-1/2}]$$

$$= \frac{15}{2} (1+3x)^{-1/2}$$

$$4. \quad x^2 - xy + y^2 = 3 \text{ at } (1, 2)$$

first derivative of equation

$$f'(x) = 2x - y = 2$$

$$2x - y = 2$$

$$y = 2x - 2$$

$$f'(x) = 2x - y$$

at point (1, 2)

$$f'(1) = 2 - y$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 2 - y(x - 1)$$

$$= -4x(1-4x^2)^{-1/2}$$

$$= \frac{-4x}{\sqrt{1-4x^2}}$$

$$h) \frac{5}{2+\sqrt{1+3x}}$$

$$= \frac{5}{2+(1+3x)^{1/2}}$$

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